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AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Sample Question Paper Version 2

Date – Morning/Afternoon Model Solutions

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \ldots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \ldots\ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \ldots\ + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ \ldots \qquad \left(|x| < 1,\ n \in \mathbb{R}\right)$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n,\ p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions

1 Simplify $\dfrac{\left(2x^2y\right)^3 \times 4x^3y^5}{2xy^{10}}$. **[2]**

$$\frac{(2x^2y)^3 \times 4x^3y^5}{2xy^{10}} = \frac{8x^6y^3 \times 4x^3y^5}{2xy^{10}} = \frac{32x^9y^8}{2xy^{10}} = \underline{\frac{16x^8}{y^2}}$$

2 Find the coefficient of x^4 in the binomial expansion of $(x-3)^5$. **[3]**

We do the binomial expansion for the x^4 term:

$$\binom{5}{1} x^4 \cdot (-3)^1 = 5x^4 \times -3 = -15x^4$$

$\Rightarrow$ x^4 coefficient is $\underline{-15}$.

3 **Fig. 3** shows a particle of weight 8 N on a rough horizontal table.
The particle is being pulled by a horizontal force of 10 N.
It remains at rest in equilibrium.

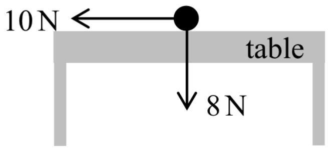


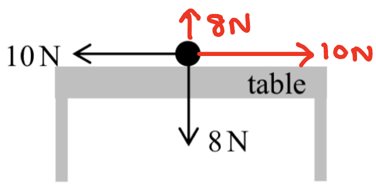
Fig. 3

(a) What information given in the question, tells you that the forces shown in **Fig. 3** cannot be the only forces acting on the particle? **[1]**

We are told that the particle is at rest in equilibrium, and we know that the forces in the diagram do not add to zero, thus there must be other forces present which are not shown.

(b) The only other forces acting on the particle are due to the particle being on the table.
State the types of these forces and their magnitudes. **[2]**

- There is 10N (to the left) of horizontal force and this will be 'matched' by 10N of friction to the right.
- There will be a normal reaction force coming from the table which will be 8N upwards.



4 **(a)** Express x^2+4x+7 in the form $(x+b)^2+c$. **[2]**

Here, we want to complete the square.

x^2+4x+7

$\div 2$

$2 \Rightarrow (x+2)^2 - 4 + 7$

square

$\Rightarrow (x+2)^2 + 3 = (x+b)^2 + c$ with $b=2$ and $c=3$.

(b) Explain why the minimum point on the curve $y=(x+b)^2+c$ occurs when $x=-b$. **[1]**

The minimum point is $x=-b$ because :

- $(x+b)^2 \geq 0$ for all x, b, which means that the minimum point will occur when $(x+b)^2=0$

$\Rightarrow (x+b)^2 = 0 \Rightarrow x+b=0 \Rightarrow x=-b$ as required.

5 Particle P moves on a straight line that contains the point O.
At time t seconds the displacement of P from O is s metres, where $s=t^3-3t^2+3$.

(a) Determine the times when the particle has zero **velocity**. **[3]**

Velocity can be found by differentiating displacement.

$S = t^3 - 3t^2 + 3 \Rightarrow V = \frac{ds}{dt} = 3t^2 - 6t$

Then we set $\frac{ds}{dt} = V = 0 \Rightarrow 3t^2 - 6t = 0 \Rightarrow 3t(t-2) = 0$

$\Rightarrow t=0$ and $t=2$ seconds

(b) Find the distances of P from O at the times when it has zero velocity. **[2]**

Recall that $s = t^3 - 3t^2 + 3$

• For $t = 0 \Rightarrow s = \underline{3m}$

• For $t = 2 \Rightarrow s = (2)^3 - 3(2)^2 + 3 = 8 - 12 + 3 = \underline{-1m}$ negative displacement but distance is still 1m.

$\Rightarrow$ distances for $t = 0$ and $t = 2$ are $\underline{3m}$ and $\underline{1m}$ respectively.

6 Two points, A and B, have position vectors $\mathbf{a} = \mathbf{i} - 3\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} + 3\mathbf{j}$.
The point C lies on the line $y = 1$. The lengths of the line segments AC and BC are equal.
Determine the position vector of C. **[4]**

$\overrightarrow{AC} = c - a = \begin{pmatrix} k \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} k-1 \\ 4 \end{pmatrix}$ then $|\overrightarrow{AC}| = \sqrt{(k-1)^2 + 16}$

$\overrightarrow{BC} = c - b = \begin{pmatrix} k \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} k-4 \\ -2 \end{pmatrix}$ then $|\overrightarrow{BC}| = \sqrt{(k-4)^2 + 4}$

The lengths are equal hence $|\overrightarrow{AC}| = |\overrightarrow{BC}|$

$\Rightarrow (k-1)^2 + 16 = (k-4)^2 + 4$

$\Rightarrow k^2 - 2k + 1 + 16 = k^2 - 8k + 16 + 4$

$\Rightarrow k^2 - 2k + 17 = k^2 - 8k + 20$

$\Rightarrow 6k = 3 \Rightarrow k = \frac{1}{2} \Rightarrow c = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$ or $c = \underline{\frac{1}{2}i + j}$

7 A car is usually driven along the whole of a 5 km stretch of road at a constant speed of 25 m s^{-1}. On one occasion, during a period of 50 seconds, the speed of the car is as shown by the speed-time graph in **Fig. 7**.
The rest of the 5 km is travelled at 25 m s^{-1}.

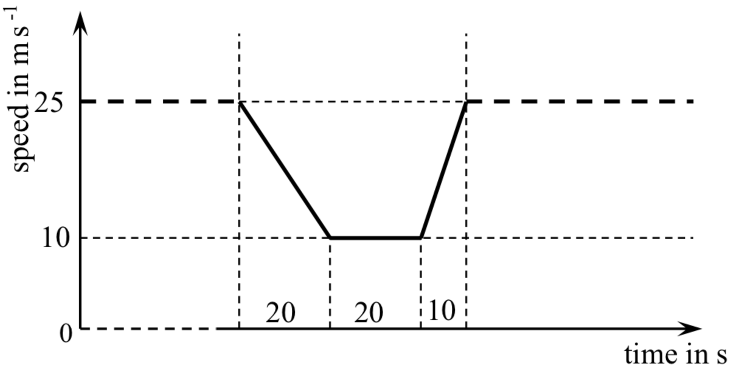


Fig. 7

How much more time than usual did the journey take on this occasion?
Show your working clearly. **[4]**

We are given a 50 second period where the speed changes.

The usual time for the whole journey will be $t = \frac{5000}{25} = 200s$.

Then the extra time will be $\frac{5000 - A}{25} + 50 - 200$

We can work out A, which will be the distance travelled in the 50s period, which will be the area under the graph :

$\Rightarrow A =$ Square - trapezium $= 50 \times 25 - \frac{20 + 50}{2} \times 15 = 725m$

$\Rightarrow$ Extra time $= \frac{5000 - 725}{25} + 50 - 200 = 21$ seconds

8 A circle has equation $(x-2)^2 + (y+3)^2 = 25$.

(a) Write down

- the radius of the circle,
- the coordinates of the centre of the circle. **[2]**

• The radius is read off the equation as the Square root of the constant, So in this case it is $\sqrt{25} = \underline{5}$.

• The center will be $(2, -3)$ as read from the equation but we swap the signs.

(b) Find, in exact form, the coordinates of the points of intersection of the circle with the y-axis. **[3]**

$(x-2)^2 + (y+3)^2 = 25$ then set $x = 0$

$\Rightarrow \quad 4 + y^2 + 6y + 9 - 25 = 0$

$\Rightarrow \quad y^2 + 6y - 12 = 0$ and we use quadratic formula:

$a = 1,\ b = 6$ and $c = -12$.

$$\Rightarrow \quad \frac{-6 \pm \sqrt{6^2 - 4(1)(-12)}}{2} = \frac{-6 \pm 2\sqrt{21}}{2}$$

$\Rightarrow \quad y = \underline{-3 + \sqrt{21}}$ and $y = \underline{-3 - \sqrt{21}}$

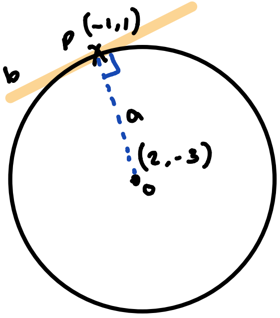
$\Rightarrow$ Coordinates are $(0, \underline{-3 + \sqrt{21}})$ and $(0, \underline{-3 - \sqrt{21}})$

(c) Show that the point (1, 2) lies outside the circle. **[2]**

We start by plugging the coordinates into the equation of Circle : $(x-2)^2+(y+3)^2+25$

$\Rightarrow (1-2)^2+(2+3)^2 = 26 > 25$ $\Rightarrow$ the point (1,2) lies outwith the circle.

(d) The point P(−1, 1) lies on the circle. Find the equation of the tangent to the circle at P. **[4]**



We first work out the gradient of a (dotted line)

$\Rightarrow m_1 = \dfrac{1-(-3)}{-1-2} = -\dfrac{4}{3}$

Then the gradient of the tangent line b is $m_2 = -\dfrac{1}{-4/3}$

$\Rightarrow m_2 = \dfrac{3}{4}$, then we can work the equation of the tangent line.

$\Rightarrow \quad y-1 = \dfrac{3}{4}(x+1)$

$\Rightarrow \quad y = \dfrac{3}{4}x + \dfrac{7}{4}$ or $4y = 3x+7$

9 A biologist is investigating the growth of bacteria in a piece of bread.
He believes that the number, N, of bacteria after t hours may be modelled by the relationship
$N = A \times 2^{kt}$, where A and k are constants.

(a) Show that, according to the model, the graph of $\log_{10} N$ against t is a straight line.

Give, in terms of A and k,
- the gradient of the line
- the intercept on the vertical axis. **[4]**

$N = A \times 2^{kt}$

$\Rightarrow \log_{10} N = \log_{10} A \times 2^{kt}$

$\Rightarrow \log_{10} N = \log_{10} A + \log_{10} 2^{kt}$

$\Rightarrow \log_{10} N = \log_{10} A + kt \log_{10} 2$

'$y = mx + c$' where $y = \log_{10} N$ and $x = t$

$\Rightarrow$ • The gradient is $\underline{\underline{k \log_{10} 2}}$ and • y intercept is $\underline{\underline{\log_{10} A}}$.

The biologist measures the number of bacteria at regular intervals over 22 hours and plots a graph of $\log_{10} N$ against t. He finds that the graph is approximately a straight line with gradient 0.20. The line crosses the vertical axis at 2.0.

(b) Find the values of A and k. **[2]**

$\log_{10} N = \log_{10} A + k \log_{10} 2t$

• $\log_{10} A = 2.0$ and $k \log_{10} 2 = 0.2$

$\Rightarrow A = 10^2 = \underline{\underline{100}}$ $\quad k = \dfrac{0.2}{\log_{10} 2} = 0.66438... = \underline{\underline{0.66}}$

(c) Use the model to predict the number of bacteria after 24 hours. **[1]**

Recall that $N = A \times 2^{kt}$ and $A = 100$, $k = 0.66$ and $t = 24$

$\Rightarrow N = 100 \times 2^{0.66 \times 24}$

$\Rightarrow N = 5865636.305$

$N = \underline{5,870,000}$ bacteria (3 sig fig)

(d) Give a reason why the model may not be appropriate for large values of t. **[1]**

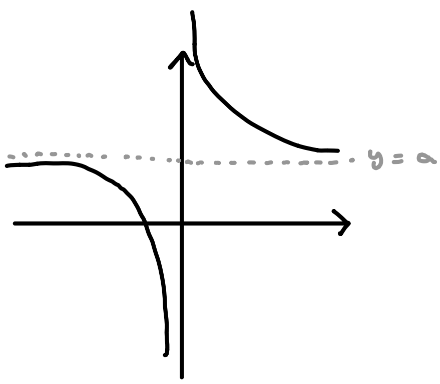
The number of bacteria on the bread may behave differently for large t as the bread will not be able to hold an infinite or very large amount of bacteria.

10 **(a)** Sketch the graph of $y = \frac{1}{x} + a$, where a is a positive constant.

- State the equations of the horizontal and vertical asymptotes.
- Give the coordinates of any points where the graph crosses the axes. **[4]**

- We want to sketch $y = \frac{1}{x}$ but we shift is up by 'a'.
- We know $y = \frac{1}{x}$ has horizontal and vertical asymptotes, and these asymptotes have equations $\underline{y = a}$ and $\underline{x = 0}$.
- The curve will cut the x-axis at $x = -\frac{1}{a}$ (found by setting $y = 0$)

$\Rightarrow$



(b) Find the equation of the normal to the curve $y = \frac{1}{x} + 2$ at the point where $x = 2$. **[5]**

$y = \frac{1}{x} + 2$ and we differentiate to get that

$\frac{dy}{dx} = -\frac{1}{x^2}$ and Subbing in $x = 2$ gives $\left.\frac{dy}{dx}\right|_2 = -\frac{1}{4}$.

The normal to this has gradient $-\frac{1}{-1/4} = \underline{4}$

and when $x = 2, y = \frac{5}{2} \Rightarrow (2, 5/2)$

The equation is found by $y - 5/2 = 4(x-2)$

$\Rightarrow y = 4x - \frac{11}{2}$ or $\underline{2y = 8x - 11}$

(c) Find the coordinates of the point where this normal meets the curve again. **[3]**

Normal : $y = 4x - \frac{11}{2}$ and Curve $y = \frac{1}{x} + 2$

$\Rightarrow 4x - \frac{11}{2} = \frac{1}{x} + 2$

$4x^2 - \frac{11}{2}x = 1 + 2x \Rightarrow 8x^2 - 11x - 4x - 2 = 0$

$\Rightarrow 8x^2 - 15x - 2 = 0$

$\Rightarrow \frac{15 \pm \sqrt{15^2 - 4(8)(-2)}}{16} = \frac{15 \pm 17}{16}$

$x = 2$ (ignore as already got this from previous question.

and $x = -\frac{1}{8} \Rightarrow y = 4\left(-\frac{1}{8}\right) - \frac{11}{2} = -6$

$\Rightarrow$ Coordinate is $\underline{(-1/8, -6)}$

11 **In this question you must show detailed reasoning.**

Determine for what values of k the graphs $y = 2x^2 - kx$ and $y = x^2 - k$ intersect. **[6]**

First, we should equate these two equations;

$\Rightarrow \quad 2x^2 - kx = x^2 - k$

$\Rightarrow \quad x^2 - kx + k = 0$ then $b^2 - 4ac = k^2 - 4k \geq 0$ since we know

And solving for K: $K(K-4) \geq 0$

$\Rightarrow$ $\underline{K \leq 0}$ and $\underline{K \geq 4}$ are the values of K such that the two graphs intersect.

12 A box hangs from a balloon by means of a light inelastic string. The string is always vertical. The mass of the box is 15 kg.

Catherine initially models the situation by assuming that there is no air resistance to the motion of the box. Use Catherine's model to calculate the tension in the string if:

(a) the box is held at rest by the tension in the string, **[1]**

The box is held at rest which means it is not accelerating.

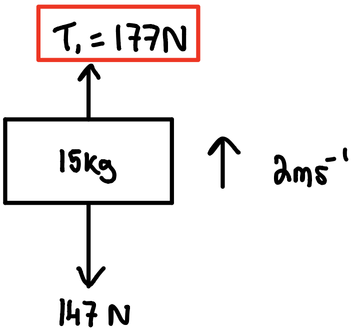
This means that the forces are balanced and the tension

will be equal to force (downward) from the mass ($F = 15 \times 9.8 = 147$ N).

Then $T = \underline{15g}$ N or $T = 15 \times 9.8 = \underline{147}$ N

(b) the box is instantaneously at rest and accelerating **upwards** at 2 m s^{-2}, **[2]**

It is useful to use a diagram here:



$F = ma = 15 \times 2 = 30N$

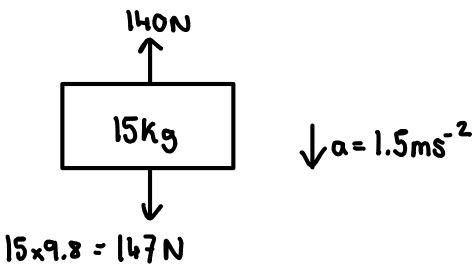
$T_1 = T + 30N = 147 + 30 = \underline{177N}$

(c) the box is moving **downwards** at 3 m s^{-1} and accelerating **upwards** at 2 m s^{-2}. **[1]**

The tension does not change, thus $T_2 = \underline{177}\ N$

Catherine now carries out an experiment to find the magnitude of the air resistance on the box when it is moving.
At a time when the box is accelerating **downwards** at 1.5 m s^{-2}, she finds that the tension in the string is 140 N.

(d) Calculate the magnitude of the air resistance at that time.
Give, with a reason, the direction of motion of the box. **[5]**



air resistance

$mg - T_3 - R = ma$

$147 - 140 - R = 15 \times 1.5$

$\Rightarrow F = \underline{-15.5}$

air resistance

The force is negative which means that it is acting in a downwards direction, and we know air resistance acts in the opposite direction to movement. This means that we can conclude that the box is moving upwards, and the magnitude of the air resistance is $\underline{15.5N}$.

END OF QUESTION PAPER